PUMPING EFFECT OF IMPELLERS WITH FLAT INCLINED BLADES*

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Hydrodynamic analysis of the flow around the blade of an impeller agitating a low-viscosity Newtonian liquid in a cylindrical baffled vessel has been employed to assess the influence of blade inclination on impeller pumping capacity. Volumetric flow rate of an impeller with flat inclined blades is understood as the sum of its axial and radial components. From the relations obtained, the ratio of individual volumetric flow rates depending on blade inclination can be calculated using a single universal parameter. Experimental results consist of the flow rate criteria values in dependence on blade inclination, for impellers with various blade numbers, in the range of Reynolds numbers $\mathrm{Re} \in \langle 1 \,.\, 10^4; 1{\cdot}5 \,.\, 10^5 \rangle$.

In mixing equipments, great diversity may be found not only in the vessel layout, but also in the design of impeller itself. The same type of impeller placed in a vessel of different shape or used in another configuration may cause a different hydrodynamic regime. Yet, the impeller design has the greatest influence on the function of a mixing equipment.

The shape of impeller and vessel depends on the purpose of mixing, which in turn is given by technological procedure. In some cases, even certain traditional designs are held to, which need not always be satisfactory with respect to mixing. Hydrodynamically, the impeller produces a convective flow of agitated charge with a particular streamline pattern and turbulence. The sum of energies of both these components of convective transfer must equal the amount of mechanical energy induced by the impeller. For the effect of mixing, convective flow as well as turbulence (characterized by a great shear stress) are important. Regarding various purposes of mixing, an optimum ratio of convective flow components and a certain turbulence level is to be attained. Consequently, it is necessary to investigate, under which conditions, for instance maximum volumetric flow rate of liquid can be achieved.

This study is focused on high-speed impellers creating a convective flow of stirred charge characterized by the volumetric flow rate of liquid through impeller[1-3,5,6].

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The resulting liquid motion in a cylindrical vessel with one mechanical impeller may be decomposed into the directions, of the axes of a rectangular coordinate system (Fig. 1). It is suitable for one axis to coincide with the impeller shaft placed in the cylindrical vessel axis. The liquid flow parallel to this axis (z) shall be named axial flow. The liquid flow in the direction of vessel radius (r) shall be termed radial and the remaining one tangential[6]. According to the ability of producing a flow in one of the mentioned directions, impellers are often classified as axial[1-4] (*e.g.* propellers), radial (*e.g.* standard turbine according to ON 691021 (ref.[4])), or tangential. For sufficient suppression of the tangential component of liquid flow it is enough to provide the vessel with suitable baffles. Yet, in order to give rise to radial and axial flow, baffles are inevitable.

Impellers with flat inclined blades (Fig. 2) are often used in chemical industry. They produce a pronounced convective flow of stirred charge. In comparison with propeller agitators, their manufacturing is less complicated. Therefore it is advantageous to replace propellers by the mentioned agitators with a certain angle of blade inclination for example according to the standard ON 691 025b (ref.[4]), in order that their significant characteristics (*e.g.* power input or pumping capacity) should be

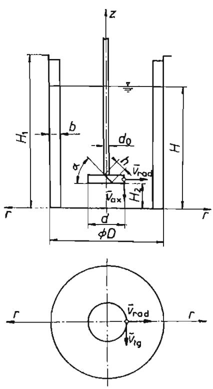

FIG. 1

Geometry of Mixing Apparatus and Schematic Representation of Liquid Flow through Impeller

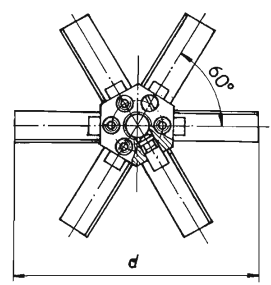

FIG. 2

Design of Impeller with Flat Inclined Adjustable Blades preserved. These impellers are often referred to as impellers with predominating axial liquid flow. It is, however, evident that with increasing blade inclination angle ($\alpha \in \langle 0°; 90° \rangle$) the ratio of radial volumetric flow rate through the impeller shall also increase. But this effect has not yet been investigated. Some authors place these impellers among the so-called mixed ones, which produce both axial and radial flow[5]. According to some studies, however, for $\alpha < 45°$ radial flow is insignificant[6,7]. Due to the fact that for blade impellers ($\alpha = 90°$) almost mere radial flow has been found in the rotor region, one may conjecture that in the range of inclination angles $\alpha \in \langle 45°; 90° \rangle$ such cases may be expected that in the flow leaving an impeller with flat inclined blades both axial and radial liquid mean velocity components may be important. Solution of this problem, however, has not been found in available sources, even though its technological significance may be interesting and useful.

THEORETICAL

The design of the mixing apparatus investigated is characterized by a flat-bottomed cylindrical vessel of diameter D (Fig. 1). The vessel is filled with a low viscosity Newtonian liquid with the still-height $H = D$. A mechanical impeller of diameter $d < D/2$ with flat inclined blades of width $h = 0{\cdot}2\ d$ is situated in the vessel axis. The inclination of impeller blades and the direction of rotation are chosen so that the axial flow rate through the impeller is directed towards the vessel bottom. Four radial baffles of the width $b = 0{\cdot}1\ D$ and of the height identical with that of the vessel, H, are placed at regular distances alongside the vessel walls. The whole investigated system is axisymmetrical, its geometry being evident from the values: $H_2/D = 0{\cdot}25$; $D/d = 3{\cdot}0$.

To determine the velocity field in the stream leaving the region of rotating impeller with flat inclined blades the following definitions (D) and simplifying assumptions (A) have to be introduced:

D *1*) The volumetric flow rate of liquid through an impeller is defined as the amount of liquid entering the rotor region of impeller, *i.e.* the volume circumscribed by the rotating impeller blades, per unit time.

D *2*) In this check volume (rotor region) neither accumulation nor source of liquid shall occur. According to the continuity equation, an incompressible liquid leaves the check volume both in axial and in radial directions. The sum of axial and radial flow rates is equal to the volume of liquid entering the check volume per unit time.

D *3*) The check area for the axial liquid flow rate is formed by the orthogonal projection of a circular area delineated by the final points of impeller blades.

D *4*) The check area for the radial liquid flow rate consists of the cylinder shell of a diameter equal to that of the impeller and of the same height as that of the projection of an inclined blade h' into the direction of rotation (Fig. 3).

A *1*) In the calculation of the check area the cross-sectional area of the hub and the final blade thickness are neglected.

A *2*) The flow-regime of agitated charge is highly turbulent and may be characterized by the Reynolds number value $Re_M > 1{\cdot}0 \,.\, 10^4$.

A *3*) The streamline pattern of agitated charge is symmetrical with respect to the impeller shaft axis.

A *4*) In the check volume the mutual interaction of impeller blades is neglected. The contributions of the individual blades to the hydraulic effect are additive.

The shape of liquid mean velocity profile in the stream leaving the rotor region through the check area for axial liquid flow[8] (Fig. 3) may be characterized by two sections: the inner one being linear, the outer one hyperbolic. The boundary between these sections is the place in which the mean velocity of the considered velocity profile ($r = r_c$) reaches its maximum value. This characterization, however, has been confirmed by a number of authors[7,9,10] even for the system studied in the present work. On condition that the liquid is given its momentum only by means of blade lift, the projections of liquid mean velocity in axial and tangential directions for an impeller with flat inclined blades may be found (Fig. 4):
For the section, where $r \leqq r_c$:

$$\bar{v}_{ax} = 2\pi nr \cos \alpha \sin \alpha \,, \tag{1}$$

$$\bar{v}_{tg} = 2\pi nr - 2\pi nr \cos^2 \alpha = 2\pi nr \sin^2 \alpha \,. \tag{2}$$

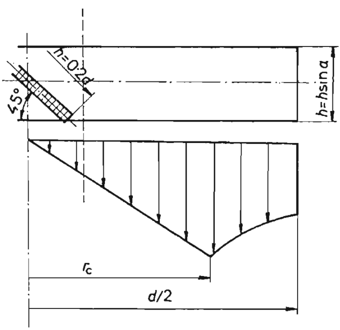

FIG. 3

Velocity Profile in Liquid Flow Leaving Rotor of Impeller with Flat Inclined Blades

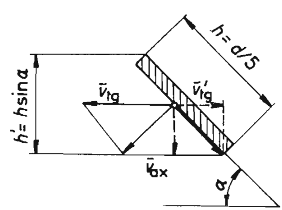

FIG. 4

Distribution of Velocities on Impeller Blade

For the section, where $r \geqq r_c$:

$$\bar{v}_{ax} = 2\pi n r c'^2 \cos\alpha \sin\alpha\,, \tag{3}$$

$$\bar{v}_{tg} = 2\pi n r c^2 \sin^2\alpha\,, \tag{4}$$

where $c = r_c/(d/2)$ and $c' = r_c/r$.

The tangential component of mean velocity thus determined is supposed to be entirely transformed into the radial one in consequence of centrifugal force:

$$\bar{v}_{tg}(r = d/2) \equiv \bar{v}_{rad}\,. \tag{5}$$

For liquid mean velocity components determined in this way, the axial volumetric flow rate of liquid can be calculated using relation

$$\dot{V}_{ax} = 2\pi \int_0^{d/2} \bar{v}_{ax}(r)\, r\, dr\,. \tag{6}$$

After substitution from relations (*1*) and (*3*) we obtain by solution and rearrangement:

$$\dot{V}_{ax} = \frac{\pi^2}{6} \cos\alpha \sin\alpha (3c^2 - 2c^3)\, nd^3\,. \tag{7}$$

Analogously, for the radial volumetric flow rate (if the ratio of blade width to impeller diameter is $h = 0{\cdot}2\,d$) one obtains:

$$\dot{V}_{rad} = \frac{\pi^2}{5} c^2 \sin^3\alpha\, nd^3\,. \tag{8}$$

Total volumetric flow rate $\dot{V}$ is given by the sum of axial and radial flow rates:

$$\dot{V} = \left[\frac{\pi^2}{6} \cos\alpha \sin\alpha (3c^2 - 2c^3) + \frac{\pi^2}{5} c^2 \sin^3\alpha\right] nd^3\,. \tag{9}$$

If, further, the flow rate criterion

$$\mathrm{Kp} = \dot{V}/nd^3 = \mathrm{Kp}_{ax} + \mathrm{Kp}_{rad} \tag{10}$$

is introduced, the final relation for the flow rate through the impeller is obtained in dimensionless form:

$$\mathrm{Kp} = \frac{\pi^2}{6} \cos \alpha \sin \alpha (3c^2 - 2c^3) + \frac{\pi^2}{5} c^2 \sin^3 \alpha \,. \qquad (11)$$

The last equation reveals the fact that the volumetric flow rate through an impeller with flat inclined blades depends on the angle of blade inclination α and on the value of parameter c. Knowing the values of this parameter, we may, from Eqs (*11*), or (*9*), calculate not only impeller pumping capacity, but also the ratio of axial and radial flow rates, for an arbitrary blade inclination angle $\alpha \in \langle 0°; 90° \rangle$.

EXPERIMENTAL

Pumping capacity has not been among commonly investigated impeller characteristics. For a critical selection of experimental method, the requirements on measuring method and equipment[8] have to be stated. For the use of planned large-scale research, the method of mean time of circulation measurements employed in this study seemed to be the most advantageous. Its theoretical basis published by Porcelli and Marr[11] proceeds from the notion of the flow in a baffled axisymmetrical system. The more detailed description of this method is, for instance, in studies already quoted[6-8]. In order to visualize the flow of agitated charge, a small tracer particle has been used, designed so as to perform motions identical with a liquid particle, whose volume it occupies within its outlines[12]. For a mean circulation time among two subsequent passes of such tracer particle through the chosen area (the rotor region of impeller) the cited authors have proposed the following relation:

$$\bar{\theta} = V/\dot{V}\,, \qquad (12)$$

where V is the known volume of agitated charge, and therefore

$$\dot{V} = V/\bar{\theta}\,. \qquad (13)$$

If the total time T of the succession of subsequent passes p_M has been measured, then

$$\bar{\theta} = T/p_M\,. \qquad (14)$$

The total number of tracer particle passes is given by the requirement of sufficient accuracy, the latter being the greater, the higher the chosen number of passes. In this study, the number of passes $p_M = 2000$ was divided into ten measuring cycles by 200 passes each, out of which the arithmetic average $\bar{\theta}$ has been calculated. The size of tracer particle defined by the radius of a circumscribed sphere (6 mm) was chosen small enough with respect to the minimum impeller diameter used ($d = 75$ mm).

The measurements were undertaken in transparent vessels of the diameter $D = 250$, 300 and 450 mm. The impellers of the diameter $d = 75$ and 150 mm were designed so as to make possible changing of blade inclination in the range $\alpha \in \langle 10°; 90° \rangle$. The impellers of the diameter $d = 100$ mm had fixed blade inclination $\alpha = 24°$ and 45°. The design of impellers with adjustable blades is illustrated by Fig. 2. All the experiments have been undertaken in the equipment driven by a motor with power input 1 000 W and a possibility of continuous change of frequency. A more detailed description can be found in study[8]. As a stirred medium water was used, its temperature maintained on 20°C.

RESULTS AND DISCUSSION

The dependence of Kp *value on the blade inclination* was experimentally investigated within the range of blade inclination angle $\alpha \in \langle 15°; 90° \rangle$. For $\alpha < 15°$ with $n_L < 4$ the impeller blades overlap at the hub, the space between them being smaller than the tracer particle diameter. Moreover, the smallest blade inclination adjustable depends on the thickness of impeller blade. The data measured are summarized in Table I. The graphical representation of the function $\mathrm{Kp} = f(\alpha)$ in Fig. 5 shows that the value of Kp increases in the blade inclination range $\alpha \in \langle 15°; 45° \rangle$. In the range $\alpha \in \langle 45°; 60° \rangle$ it slightly changes and for $\alpha \in \langle 60°; 90° \rangle$ it decreases. The data obtained in this way make possible to calculate the value of parameter c by means of inverse solution of Eq. (*11*). A typical course of function $c = f(\alpha)$ for the impeller with blade number $n_L = 6$ is in Fig. 6. The values of $c = f(\alpha)$ for the

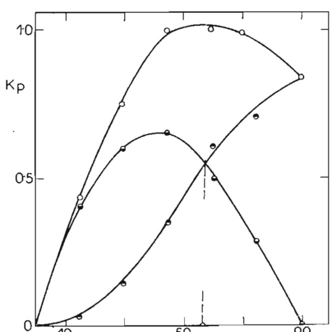

FIG. 5

Dependence of Kp, $\mathrm{Kp_{ax}}$, $\mathrm{Kp_{rad}}$ Values on Blade Inclination Angle. Impeller with Flat Inclined Blades ($n_L = 6$)
Kp ○; $\mathrm{Kp_{ax}}$ ◒; $\mathrm{Kp_{rad}}$ ◓.

other impellers ($n_L = 2$; 3 and 4) are summed up in Table II. Apparently, for $\alpha > 30°$ the c value may be considered constant. Knowing the parameter c, the ratio of axial and radial liquid flow rates through the impeller may be calculated from Eq. (*11*). The graphical representation of these calculation results for the impeller with the blade number $n_L = 6$ is shown in Fig. 5. For the other investigated impellers the course is analogous and easily calculable by means of the c values shown in Table II.

The dependence of flow rate criterion Kp *on the blade inclination angle* is given by Eq. (*11*). As the axial and radial liquid flow rates through the impeller are supposed, it seems useful to follow the respective flow rates in the discussion.

Besides the blade inclination angle, the total volumetric flow rate depends on the parameter c, its value lying in the interval $c \in \langle 0; 1 \rangle$. Thus the basic assumption on the course of liquid velocity profile along impeller blade is fulfilled (Fig. 4). For angles $\alpha > 40°$ the value of c may be considered constant and independent of blade

TABLE I

Dependence of Flow Rate Criterion Kp on Blade Inclination for Various Blade Numbers n_L

α	Kp			
	$n_L = 2$	$n_L = 3$	$n_L = 4$	$n_L = 6$
15°	0·2879	0·3315	0·3510	0·4370
30°	0·5523	0·6452	0·7061	0·7470
40°	unmeasured	0·7546	unmeasured	0·8793
45°	0·7125	0·7822	0·8502	0·9937
50°	unmeasured	0·8387	unmeasured	1·0260
60°	0·7775	0·8434	0·9433	1·0070
75°	unmeasured	0·8373	unmeasured	0·9860
90°	0·6242	0·7420	0·7853	0·8380

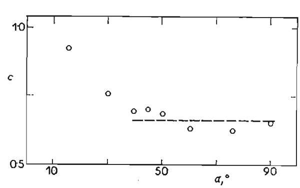

FIG. 6

Dependence of Parameter c on Inclination Angle of Impeller Blade ($n_L = 6$)

inclination for all impellers investigated (Table II). Introducing this into Eq. (*11*) we are able to determine the maximum axial flow rate using the Eq. (*15*):

$$\mathrm{d}Kp_{ax}/\mathrm{d}\alpha = 0 \tag{15}$$

for $\alpha = 45°$.

Analogously for

$$\mathrm{d}Kp_{rad}/\mathrm{d}\alpha = 0 \tag{16}$$

the maximum value will be for $\alpha = 90°$.

To verify these results, the axial and radial volumetric flow rates ought to be measured separately, which is impossible, in terms of the method employed, without the adaptation of the impeller. To make at least a qualitative assessment, an adapted impeller with flat inclined blades was developed, provided with the so-called circumferential ring (Fig. 7). The purpose of this adaptation was to attain a minimum radial liquid flow rate. The impeller design was adapted so that the blades could be turned in the same range of blade inclination angle ($\alpha \in \langle 15°; 90° \rangle$) as in the case of previous investigations. The comparison of flow rate criteria values of the impeller thus adapted and the impeller with flat inclined blades (Fig. 8) shows that for $\alpha < 30°$, the circumferential ring does not influence the volumetric flow rate of liquid, which is mainly axial. For an angle $\alpha < 30°$, the influence of circumferential ring has already been apparent. Suppose the difference of the Kp values thus obtained is

TABLE II

Parameter c Values in Dependence on Blade Inclination for Various Blade Numbers n_L

α	c			
	$n_L = 2$	$n_L = 3$	$n_L = 4$	$n_L = 6$
15°	0·6150	0·6870	0·7200	0·9200
30°	0·6020	0·6730	0·7220	0·7560
40°	unmeasured	0·6225	unmeasured	0·6930
45°	0·5640	0·5990	0·6330	0·7030
50°	unmeasured	0·5975	unmeasured	0·6810
60°	0·5420	0·5685	0·6080	0·6325
75°	unmeasured	0·5740	unmeasured	0·6285
90°	0·5626	0·6136	0·6313	0·6516

proportional to the radial volumetric flow rate of liquid, this difference must be maximum for $\alpha = 90°$, which is apparent from Fig. 8. As results from the theoretical reasoning based on Eq. (*16*), however, for $\alpha = 90°$ the radial volumetric flow rate should coincide with the total flow rate. The discord between the experimental data in Fig. 8 and this consideration may be explained by the influence of final vessel size. The impeller does not work in an unlimited environment and the radial liquid flow caused by impeller with flat inclined blades turns in at the vessel walls. Thus, in fact, it supports the axial volumetric flow rate. In steady state, the liquid enters the check volume with some axial velocity component, even if the impeller blade inclination $\alpha = 90°$. This is also why in the studied impeller type for $\alpha = 90°$ the upper and lower circulation circuits do not appear as in the case of classic disc turbine impellers (*e.g.* according to the ON 691 021 (ref.[4]) standard). The circumferential ring never entirely prevents the manifestation of radial component, especially for great blade inclination angles, when its width approaches to the projection of inclined blade h' into the direction of rotation. In addition to this, the placing of the circumferential ring around the impeller will change its geometry, causing even for instance the rise of power input for small blade inclination angles. There-

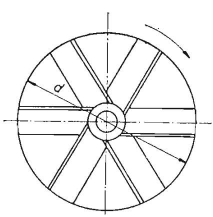

FIG. 7

Impeller with Circumferential Ring

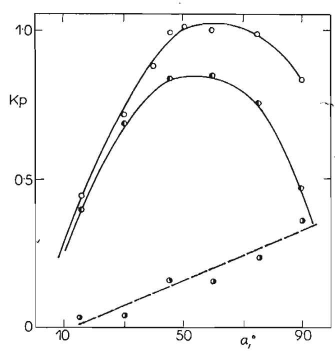

FIG. 8

Comparison of Kp Values for Impellers with Flat Inclined Blades and Impellers with Circumferential Ring ($n_L = 6$)

Impeller with circumferential ring ◐; impeller with flat inclined blades ○; difference of Kp values ◑.

fore the data in Fig. 8 ought to be considered only informative, proving a mere existence of radial liquid flow rate in impellers with flat inclined blades.

The shape of velocity profile of the liquid stream leaving the check volume, is characterized by the universal parameter c (Eq. (*11*)). The purpose of the study was to investigate, why the parameter increases for $\alpha < 30°$; it does not correspond to the suggested theoretical conceptions: the c values increase disproportionately for $\alpha < 30°$. This fact may be explained so that for small blade inclination angles even the manifestation of the lift component becomes significant as a result of liquid circulation around the impeller blade[13].

The plane sheet forming the angle α with the direction of motion is affected not only by the drag, but also by the lift perpendicular to the direction of motion. The impeller blade may be considered a plane sheet performing a rotational motion. In the Theoretical the liquid circulation around a blade has not been taken into account, while the liquid was presumed to obtain all the momentum by means of lift. The experimental method used was suitable only for determining the total flow rate value, including also the increase of lift component. This is the reason why the parameter c must increase for $\alpha < 30°$.

This fact may be verified for instance by creating impeller blades of a profile known to have greater lift effect (*e.g.* curved blades, aerofoil wing profile *etc.*) than a flat inclined blade. As the impeller with aerofoil wing blades was not available, the impellers with curved blades formed by a section of cylinder lateral area of the radius $r_z = 0{\cdot}25\ d$ were experimentally tested. Fig. 9 shows a graphical representation of such impeller data found experimentally and compared to the Kp values for the impeller with flat inclined blades. Apparently, for $\alpha = 15°$ the impeller with

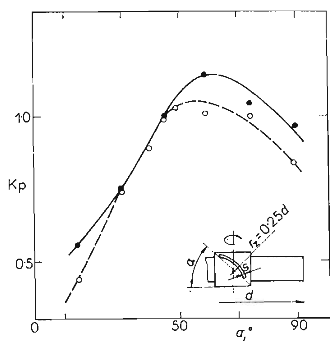

FIG. 9

Comparison of Kp Values of Impellers with Flat Inclined Blades and Impellers with Curved Blades ($n_L = 6$)

Impeller with flat inclined blades ○; impeller with curved blades ●.

curved blades ($n_L = 6$) has the Kp value by 27% higher. For $\alpha = 30°$ and 45° a significant Kp value increase has not been found. For $\alpha > 45°$, again, the Kp value increase is apparent as a result of a greater rate of radial liquid flow component.

CONCLUSION

The results of investigations have shown that impellers with flat inclined blades display both axial and radial components of volumetric flow rate. For blade inclination angles $\alpha < 30°$, the radial flow rate may be considered insignificant. For the blade inclination angle $\alpha = 45°$, which was mostly used in the studied impellers, however, radial volumetric flow rate has become significant. Due to the final vessel size, partial axial flow of agitated charge appears even if $\alpha = 90°$.

LIST OF SYMBOLS

b baffle width (m)
c $r_c/(d/2)$
c' r_c/r
D vessel diameter (m)
d impeller diameter (m)
H still height of liquid in vessel (m)
H_1 total height of vessel (m)
H_2 distance of impeller lower edge from vessel bottom (m)
h impeller blade width (m)
n impeller rotation frequency (s^{-1})
n_L impeller blade number
p_M number of tracer particle passes through check area
r radius (m)
r_c limiting radius according to Eqs (*3*) and (*4*) (m)
T total time of tracer particle circulations (s)
V total volume of stirred charge (m^3)
$\dot{V}$ impeller pumping capacity ($m^3 s^{-1}$)
$\bar{v}$ liquid mean velocity ($m s^{-1}$)
α inclination angle of impeller blade (°)
$\bar{\theta}$ mean time of circulation (s)
η dynamic viscosity ($kg m^{-1} s^{-1}$)
ϱ density ($kg m^{-3}$)
Kp $= \dot{V}n^{-1}d^{-3}$ flow rate criterion for mixing
$Re_M = d^2n^1 \varrho\eta^{-1}$ Reynolds number

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